### A.3.2.2 Simplified procedure for fixed base cylindrical tanks [6]

#### A.3.2.2.1 Model

The tank-liquid system is modeled by two single-degree-of-freedom systems, one corresponding to the impulsive component, moving together with the flexible wall, and the other corresponding to the convective component. The impulsive and convective responses are combined by taking their numerical-sum.

The natural periods of the impulsive and the convective responses, in seconds, are taken as:

$$T_{\mathrm{imp}} = C_{\mathrm{i}} \frac{\sqrt{\rho}\, H}{\sqrt{\mathrm{s/R}}\, \sqrt{\mathrm{E}}} \tag{A.35}$$

$$T_{\mathrm{con}} = C_{\mathrm{c}} \sqrt{R} \tag{A.36}$$

where:

$H$ = height to the free surface of the liquid;

$R$ = tank's radius;

$s$ = equivalent uniform thickness of the tank wall (weighted average over the wetted height of the tank wall, the weight may be taken proportional to the strain in the wall of the tank, which is maximum at the base of the tank);

$\rho$ = mass density of liquid; and

$E$ = Modulus of elasticity of tank material.

**Table A.2** — Coefficients $C_i$ and $C_c$ for the natural periods, masses $m_i$ and $m_c$ and heights $h_i$ and $h_c$ from the base of the point of application of the wall pressure resultant, for the impulsive and convective components

| $H/R$ | $C_{\mathrm{i}}$ | $C_{\mathrm{c}}$ (s/m$^{1/2}$) | $m_i/m$ | $m_c/m$ | $h_i/H$ | $h_c/H$ | $h'_i/H$ | $h'_c/H$ |
|---|---|---|---|---|---|---|---|---|
| 0,3 | 9,28 | 2,09 | 0,176 | 0,824 | 0,400 | 0,521 | 2,640 | 3,414 |
| 0,5 | 7,74 | 1,74 | 0,300 | 0,700 | 0,400 | 0,543 | 1,460 | 1,517 |
| 0,7 | 6,97 | 1,60 | 0,414 | 0,586 | 0,401 | 0,571 | 1,009 | 1,011 |
| 1,0 | 6,36 | 1,52 | 0,548 | 0,452 | 0,419 | 0,616 | 0,721 | 0,785 |
| 1,5 | 6,06 | 1,48 | 0,686 | 0,314 | 0,439 | 0,690 | 0,555 | 0,734 |
| 2,0 | 6,21 | 1,48 | 0,763 | 0,237 | 0,448 | 0,751 | 0,500 | 0,764 |
| 2,5 | 6,56 | 1,48 | 0,810 | 0,190 | 0,452 | 0,794 | 0,480 | 0,796 |
| 3,0 | 7,03 | 1,48 | 0,842 | 0,158 | 0,453 | 0,825 | 0,472 | 0,825 |

The coefficients $C_i$ and $C_c$ are obtained from Table A.2. Coefficient $C_i$ is dimensionless, while if $R$ is in meters $C_c$ is expressed in s/m$^{1/2}$.

The impulsive and convective masses $m_i$ and $m_c$ are given in Table A.2 as fractions of the total liquid mass $m$, along with the heights from the base of the point of application of the resultant of the impulsive and convective hydrodynamic wall pressure, $h_i$ and $h_c$.

### A.3.2.2.2 Seismic response

The total base shear is

$$Q = (m_i + m_w + m_r) S_e(T_{imp}) + m_c S_e(T_{con}) \tag{A.37}$$

where:

$m_w$ = mass of the tank wall;

$m_r$ = mass of tank roof;

$S_e(T_{imp})$ = impulsive spectral acceleration, obtained from an elastic response spectrum for a value of damping consistent with the limit state considered according to **2.3.3.1**;

$S_e(T_{con})$ = convective spectral acceleration, from a 0,5%-damped elastic response spectrum.

The overturning moment immediately above the base plate is

$$M = (m_i h_i + m_w h_w + m_r h_r) S_e(T_{imp}) + m_c h_c S_e(T_{con}) \tag{A.38}$$

$h_w$ and $h_r$ are heights of the centres of gravity of the tank wall and roof, respectively.

The overturning moment immediately below the base plate is given by

$$M' = (m_i h'_i + m_w h_w + m_r h_r) S_e(T_{imp}) + m_c h'_c S_e(T_{con}) \tag{A.39}$$

The vertical displacement of liquid surface due to sloshing is given by expression (A.15).

### A.3.3 Vertical component of the seismic action

In addition to the pressure $p_{vr}(\varsigma,t)$ given by expression (A.17), due to the tank moving rigidly in the vertical direction with acceleration $A_v(t)$, there is a contribution to the pressure, $p_{vf}(\varsigma,t)$, due to the deformability (radial 'breathing') of the shell [7]. This additional term may be calculated as:

$$p_{vf}(\varsigma,t) = 0{,}815 f(\gamma) \rho H \cos\left(\frac{\pi}{2}\varsigma\right) A_{vf}(t) \tag{A.40}$$

where:

$f(\gamma) = 1{,}078 + 0{,}274 \ln\gamma$ for $0{,}8 \leq \gamma < 4$ (A.41a)

$f(\gamma) = 1{,}0$ for $\gamma < 0{,}8$ (A.41b)

$A_{vf}(t)$ is the acceleration response of a simple oscillator having a frequency equal to the fundamental frequency of the axisymmetric vibration of the tank with the fluid.

The fundamental frequency may be estimated from the expression:

$$f_{\mathrm{vd}} = \frac{1}{4R}\left[\frac{2EI_1(\gamma_1)s(\varsigma)}{\pi\rho H(1-\nu^2)I_o(\gamma_1)}\right]^{1/2} \quad \text{(for } \varsigma = 1/3\text{)} \tag{A.42}$$

where:

$\gamma_1 = \pi/(2\gamma)$;

$I_0(\cdot)$ and $I_1(\cdot)$ denote the modified Bessel function of order 0 and 1, respectively;

$E$ and ν are Young's modulus and Poisson ratio of the tank material, respectively.

The maximum value of $p_{\mathrm{vf}}(t)$ is obtained from the vertical acceleration response spectrum for the appropriate values of period and damping. If soil flexibility is neglected (see **A.7**) the applicable damping values are those of the material of the shell. The behaviour factor value, $q$, adopted for the response due to the impulsive component of the pressure and the tank wall inertia may be used for the response to the vertical component of the seismic action. The maximum value of the pressure due to the combined effect of $p_{\mathrm{vr}}(\cdot)$ and $p_{\mathrm{vf}}(\cdot)$ may be obtained by applying the 'square root of the sum of squares' rule to the individual maxima.

### A.3.4 Combination of the effects of the horizontal and vertical components of the seismic action, including the effects of other actions

The pressure on the tank walls should be determined in accordance with **A.2.3.**

## A.4 Rectangular tanks

### A.4.1 Rigid rectangular tanks on-ground, fixed to the foundation

For tanks with walls assumed as rigid, the total pressure is again given by the sum of an impulsive and a convective contribution:

$$p(z,t) = p_{\mathrm{i}}(z,t) + p_{\mathrm{c}}(z,t) \tag{A.43}$$

The impulsive component follows the expression:

$$p_{\mathrm{i}}(z,t) = q_o(z)\rho L A_{\mathrm{g}}(t) \tag{A.44}$$

where:

$L$ is the <u>half-width</u> of the tank in the direction of the seismic action;

$q_{\mathrm{o}}(z)$ is a function giving the variation of $p_{\mathrm{i}}(\cdot)$ along the height as plotted in Figure A.5 ($p_{\mathrm{i}}(\cdot)$ is constant in the direction orthogonal to the seismic action). The trend and the numerical values of $q_{\mathrm{o}}(z)$ are very close to those of a cylindrical tank with radius $R = L$ (see Figure A.6).

The convective pressure component is given by a summation of modal terms (sloshing modes). As for cylindrical tanks, the dominant contribution is that of the fundamental mode:

$$p_{\mathrm{c1}}(z,t) = q_{\mathrm{c1}}(z)\rho L A_1(t) \tag{A.45}$$

where

$q_{c1}(z)$ is a function shown in Figure A.7 together with the 2nd mode contribution $q_{c2}(z)$ and

$A_1(t)$ is the acceleration response function of a simple oscillator with the frequency of the first mode and the appropriate value of damping, when subjected to an input acceleration $A_g(t)$.

The period of oscillation of the first sloshing mode is:

$$T_1 = 2\pi \left( \frac{L/g}{\frac{\pi}{2}\tanh\left(\frac{\pi}{2}\frac{H}{L}\right)} \right)^{1/2} \tag{A.46}$$

The base shear and the moment on the foundation may be evaluated on the basis of expressions (A.44) and (A.45). The values of the masses $m_i$ and $m_{c1}$, as well as of the corresponding heights above the base, $h_i$ and $h_{c1}$, calculated for cylindrical tanks and given by expressions (A.4), (A.12) and (A.6), (A.14), respectively, may be adopted for the design of rectangular tanks as well (with $L$ replacing $R$), with an error less than 15% [8].

### A.4.2 Flexible rectangular tanks on-ground, fixed to the foundation

As in cylindrical tanks with circular section, wall flexibility generally produces a significant increase of the impulsive pressures, while leaving the convective pressures practically unchanged. Studies on the seismic response of flexible rectangular tanks are few and their results are not in a form suitable for direct use in design [9]. An approximation for design purposes is to use the same vertical pressure distribution as for rigid walls [8], see expression (A.44) and Figures A.5, A.6, but to replace the ground acceleration $A_g(t)$ in expression (A.44) with the response acceleration of a simple oscillator having the frequency and the damping ratio of the first impulsive tank-liquid mode.

This period of vibration may be approximated as:

$$T_f = 2\pi (d_f / g)^{1/2} \tag{A.47}$$

where:

$d_f$ is the deflection of the wall on the vertical centre-line and at the height of the impulsive mass, when the wall is loaded by a load uniform in the direction of the ground motion and of magnitude: $m_i g/4BH$;

$2B$ is the tank width perpendicular to the direction of the seismic action.

The impulsive mass $m_i$ may be obtained as the sum of that from expression (A.4), Figure A.2(a) or column 4 in Table A.2, plus the wall mass.

### 2.3.3 Damping

#### 2.3.3.1 Structural damping

(1) If the damping values are not obtained from specific information, the following values of the damping ratio should be used in linear analysis:

a) damage limitation state: the values specified in EN 1998-2:2005, **4.1.3(1)**;

b) ultimate limit state: $\xi = 5\%$

#### 2.3.3.2 Contents damping

(1) The value $\xi = 0{,}5$ % may be adopted for the damping ratio of water and other liquids, unless otherwise determined.

NOTE: Reference to additional information for the determination of damping ratios of liquids is given in Informative Annex B.

(2) For granular materials an appropriate value for the damping ratio should be used. In the absence of more specific information a value of $\xi = 10\%$ may be used.

#### 2.3.3.3 Foundation damping

(1) Material damping varies with the nature of the soil and the intensity of shaking. When more accurate determinations are not available, the values given in Table 4.1 of EN 1998-5: 2004 should be used.

(2)P Radiation damping depends on the direction of motion (horizontal translation, vertical translation, rocking, etc..), on the geometry of the foundation, on soil layering and soil morphology. The values adopted in the analysis shall be compatible with actual site conditions and shall be justified with reference to acknowledged theoretical and/or experimental results. The values of the radiation damping used in the analysis shall not exceed a maximum value $\xi_{max}$.

NOTE: The value to be ascribed to $\xi_{max}$ for use in a country may be found in its National Annex. Guidance for the selection and use of damping values associated with different foundation motions is provided in EN 1998-6:2005. The recommended value is $\xi_{max} = 25\%$.

#### 2.3.3.4 Weighted damping

(1) The global average damping of the whole system should account for the contributions of the different materials/elements to damping.

NOTE Procedures for accounting for the contributions of the different materials/elements to the global average damping of the system are presented in EN 1998-2:2005, **4.1.3(1),** Note and in EN 1998-6:2005, Informative Annex B.

### 2.4 Behaviour factors

(1)P For the damage limitation state, the behaviour factor $q$ shall be taken as equal to 1,0.

NOTE: For structures covered by this standard significant energy dissipation is not expected for the

$$h_{cn} = H\left(1+\frac{1-\cosh(\lambda_n \gamma)}{\lambda_n \gamma \sinh(\lambda_n \gamma)}\right) \qquad \text{(A.14b)}$$

The convective component of the response may be obtained from that of oscillators having masses $m_{cn}$, attached to the rigid tank through springs having stiffnesses: $K_n = \omega_n^2 m_{cn}$. (one oscillator for each mode considered significant, normally only the first one). The tank is subjected to the ground acceleration time-hisory $A_g(t)$ and the masses respond with accelerations $A_{cn}(t)$. $h'_{cn}$ or $h_{cn}$ is the level where the oscillator needs to be applied in order to give the correct value of $M'_{cn}$ or $M_{cn}$, respectively.

### A.2.1.4 Height of the convective wave

The sloshing wave height is provided mainly by the first mode; the expression for the peak height at the edge is:

$$d_{max} = 0{,}84 R S_e(T_{c1})/g \qquad \boxed{\text{(A.15)}}$$

where $S_e(\cdot)$ is the elastic response spectral acceleration at the 1st convective mode of the fluid for damping a value appropriate for the sloshing response and $g$ is the acceleration of gravity.

### A.2.1.5 Effect of the inertia of the walls

For steel tanks, the inertia forces on the shell due to its own mass are small compared with the hydrodynamic forces and may be neglected. For concrete tanks, they should not be neglected. Inertia forces are parallel to the horizontal seismic action, inducing a pressure normal to the surface of the shell given by:

$$p_w = \rho_s s(\varsigma) \cos\theta A_g(t) \qquad \text{(A.16)}$$

where:

$\rho_s$ = mass density of the wall material

$s(\varsigma)$ = wall thickness

The action effects of this pressure component, which follows the variation of wall thickness along the height, should be added to those of the impulsive component given by expression (A.1).

The total shear at the base due to the inertia forces of the tank wall and roof may be taken equal to the total mass of the tank walls and roof, times the acceleration of the ground. The contribution to the base overturning moment in a similar way: it is equal to the wall mass times the wall midheight (for constant wall thickness), plus the roof mass times its mean distance from the base, times the acceleration of the ground.

### A.2.1.6 Combination of action effects of impulsive and convective pressures

The time-history of the total pressure is the sum of the following two time-histories:

– the impulsive one being driven by $A_g(t)$ (including the inertia of the walls);

– the convective one driven by $A_{c1}(t)$ (neglecting higher order components).

- an impulsive mass $m_i$ rigidly connected to the tank walls, located at a height $h'_i$ or $h_i$ above the tank bottom (expressions (A.4) and (A.6a), (A.6b), respectively);

- a mass $m_{c1}$, connected to the walls through a spring of stiffness $K_{c1} = \omega^2_{c1} m_{c1}$, where $\omega_{c1}$ is given by expression (A.9), located at a height $h'_{c1}$ or $h_{c1}$ (expressions (A.12) and (A.14a), (A.14b), respectively).

The response of the system may be evaluated using standard modal analysis and response spectra methods.

In the simplest case, the global model has only two degrees-of-freedom, corresponding to the masses $m_i$ and $m_{c1}$. A mass $\Delta m$ equal to the mass of the tank and an appropriate portion of the mass of the support should be added to $m_i$. The mass ($m_i + \Delta m$) should be connected to the ground by a spring representing the stiffness of the support.

Normally, the rotational inertia of the mass ($m_i + \Delta_m$), and the corresponding additional degree of freedom, should also be included in the model.

Elevated tank in the shape of a truncated inverted cone may be considered in the model as an equivalent cylinder of the same volume of liquid and a diameter equal to that of the cone at the level of the liquid.

## A.7 Soil-structure interaction effects for tanks on-ground

### A.7.1 General

For tanks founded on relatively deformable soils, the base motion can be significantly different from the free-field motion; in general the translational component is modified and there is also a rocking component. Moreover, for the same input motion, as the flexibility of the ground increases, the fundamental period of the tank-fluid system and the total damping increase, reducing the peak force response. The increase in the period is more pronounced for tall, slender tanks, because the contribution of the rocking component is greater. The reduction of the peak force response, however, is in general less for tall tanks, since the damping associated with rocking is smaller than that associated with horizontal translation.

A simple procedure, proposed for buildings in [10] and consisting of an increase of the fundamental period and of the damping of the structure, which is considered to rest on a rigid soil and subjected to the free-field motion, has been extended to the impulsive (rigid and flexible) components of the response of tanks in [11], [12], [13]. The convective periods and pressures are assumed not to be affected by soil-structure interaction. A good approximation can be obtained through the use of an equivalent simple oscillator with parameters adjusted to match frequency and peak response of the actual system. The properties of this substitute oscillator are given in [11], [13] in the form of graphs, as functions of the ratio $H/R$, for fixed values of the wall thickness ratio $s/R$, the initial damping, etc.

### A.7.2 Simple procedure

#### A.7.2.1 Introduction

A more rough procedure [8], summarized below, may be adopted. The procedure operates by changing separately the frequency and the damping of the impulsive rigid and the impulsive flexible pressure contributions in **A.2** to **A.5.** In particular, for the rigid impulsive pressure components, whose time-histories are given by the free-field horizontal, $A_g(t)$, and vertical, $A_v(t)$ accelerations, consideration of soil-structure interaction effects amounts to replacing these time-histories with the response acceleration histories of a single degree of freedom oscillator having natural period and damping as specified below.

**A.7.2.2 Modified natural periods**:

– ‘rigid tank’ impulsive effect, horizontal

$$T_i^* = 2\pi\left(\frac{m_i + m_o}{k_x \alpha_x} + \frac{m_i h_i'^2}{k_\theta \alpha_\theta}\right)^{1/2} \tag{A.52}$$

– ‘deformable tank’ impulsive effect, horizontal

$$T_f^* = T_f\left(1 + \frac{k_f}{k_x \alpha_x}\cdot\left[1 + \frac{k_x h_f^2}{k_\theta \alpha_\theta}\right]\right)^{1/2} \tag{A.53}$$

– ‘rigid tank’, vertical

$$T_{vr}^* = 2\pi\left(\frac{m_{\text{tot}}}{k_v \alpha_v}\right)^{1/2} \tag{A.54}$$

– "deformable tank”, vertical

$$T_{vd}^* = T_{vd}\left(1 + \frac{k_l}{k_v \alpha_v}\right)^{1/2} \tag{A.55}$$

where:

$m_i$ , $h'_i$ are the mass and height of the impulsive component;

$m_o$ is the mass of the foundation;

$k_f$ is the stiffness of the "deformable tank" = $4\pi^2 \dfrac{m_f}{T_f^2}$;

$m_{tot}$ is the total mass of the filled tank, including the foundation;

$k_l$ $= 4\pi^2 \dfrac{m_l}{T_{vd}^2}$, with $m_l$ = mass of the liquid;

$k_x$, $k_\theta$, $k_v$ are the horizontal, rocking and vertical stiffness of the foundation; and

$\alpha_x$, $\alpha_\theta$, $\alpha_v$ are frequency-dependent factors converting static stiffnesses into dynamic ones [14].

**A.7.2.3 Modified damping values:**

The general expression for the effective damping ratio of the tank-foundation system is:

$$\xi = \xi_{\mathrm{s}} + \frac{\xi_{\mathrm{m}}}{\left(T^{*}/T\right)^{3}} \tag{A.56}$$

where:

$\xi_{\mathrm{s}}$ is the radiation damping in the soil; and

$\xi_{\mathrm{m}}$ is the material damping in the tank.

Both $\xi_{\mathrm{s}}$ and $\xi_{\mathrm{m}}$ depend on the specific vibration mode.

In particular for $\xi_{\mathrm{s}}$:

– for the horizontal impulsive 'rigid tank' mode:

$$\xi_{s} = \frac{2\pi^{2} m_{i}}{k_{x} T_{i}^{*2}} a \left( \frac{\beta_{x}}{\alpha_{x}} + \frac{k_{x} h'^{2}_{i} \beta_{\theta}}{k_{\theta} \alpha_{\theta}} \right) \tag{A.57}$$

– for the horizontal impulsive 'deformable tank' mode:

$$\xi_{\mathrm{s}} = \frac{2\pi^{2} m_{\mathrm{f}}}{k_{\mathrm{x}} T_{\mathrm{f}}^{*2}} a \left( \frac{\beta_{\mathrm{x}}}{\alpha_{\mathrm{x}}} + \frac{k_{\mathrm{x}} h_{\mathrm{f}}^{2} \beta_{\theta}}{k_{\theta} \alpha_{\theta}} \right) \tag{A.58}$$

– for the vertical 'rigid tank' mode:

$$\xi_{s} = \frac{2\pi^{2} m_{tot}}{k_{v} T_{vr}^{*2}} a \frac{\beta_{v}}{\alpha_{v}} \tag{A.59}$$

where:

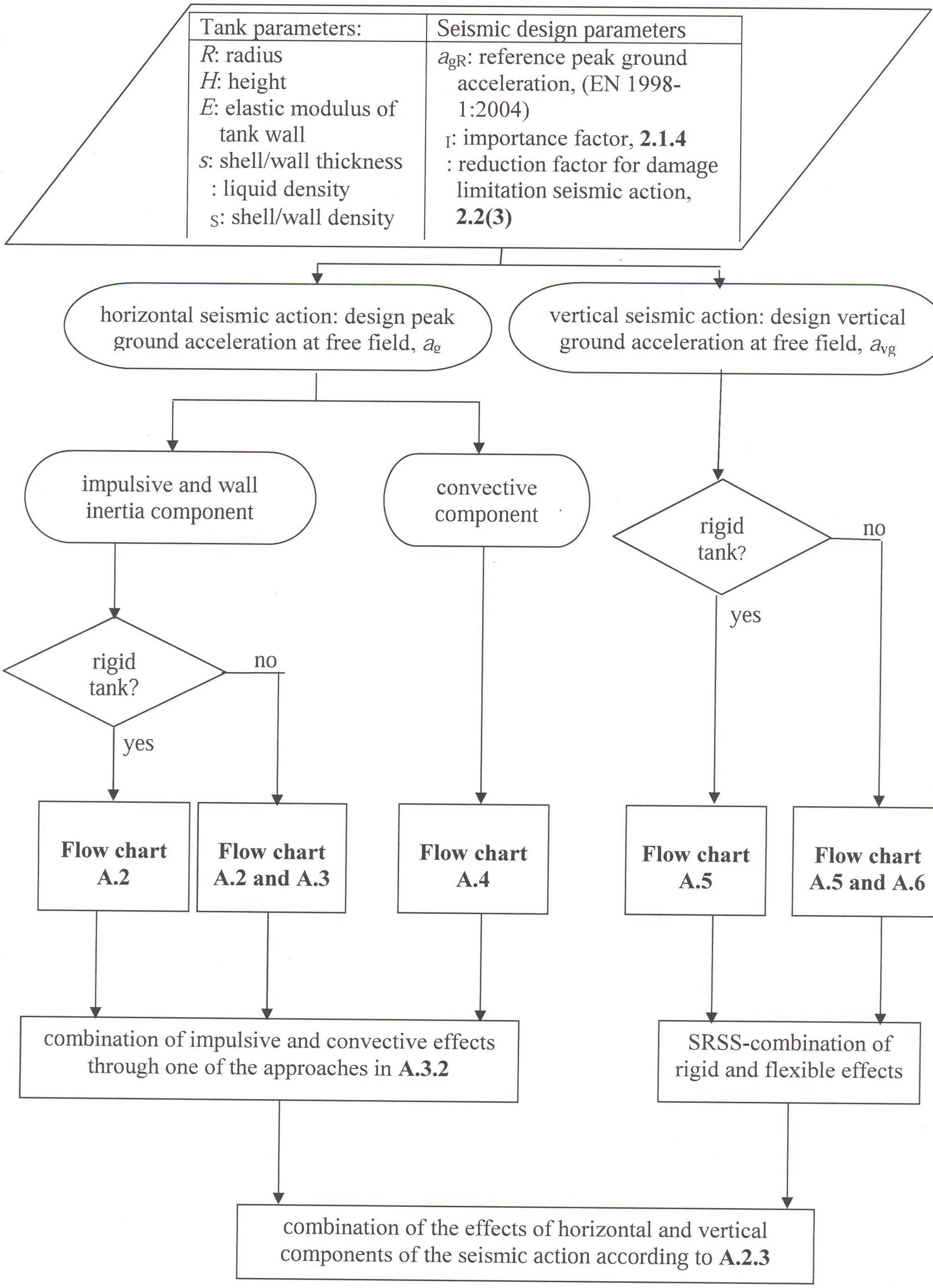
$a$ is the dimensionless frequency function = $\dfrac{2\pi R}{V_{\mathrm{s}} T}$ ($V_{\mathrm{s}}$ = shear wave velocity of the soil);

$\beta_{\mathrm{x}}, \beta_{\theta}, \beta_{v}$ are the frequency-dependent factors providing radiation damping values for horizontal, vertical and rocking motions [14].

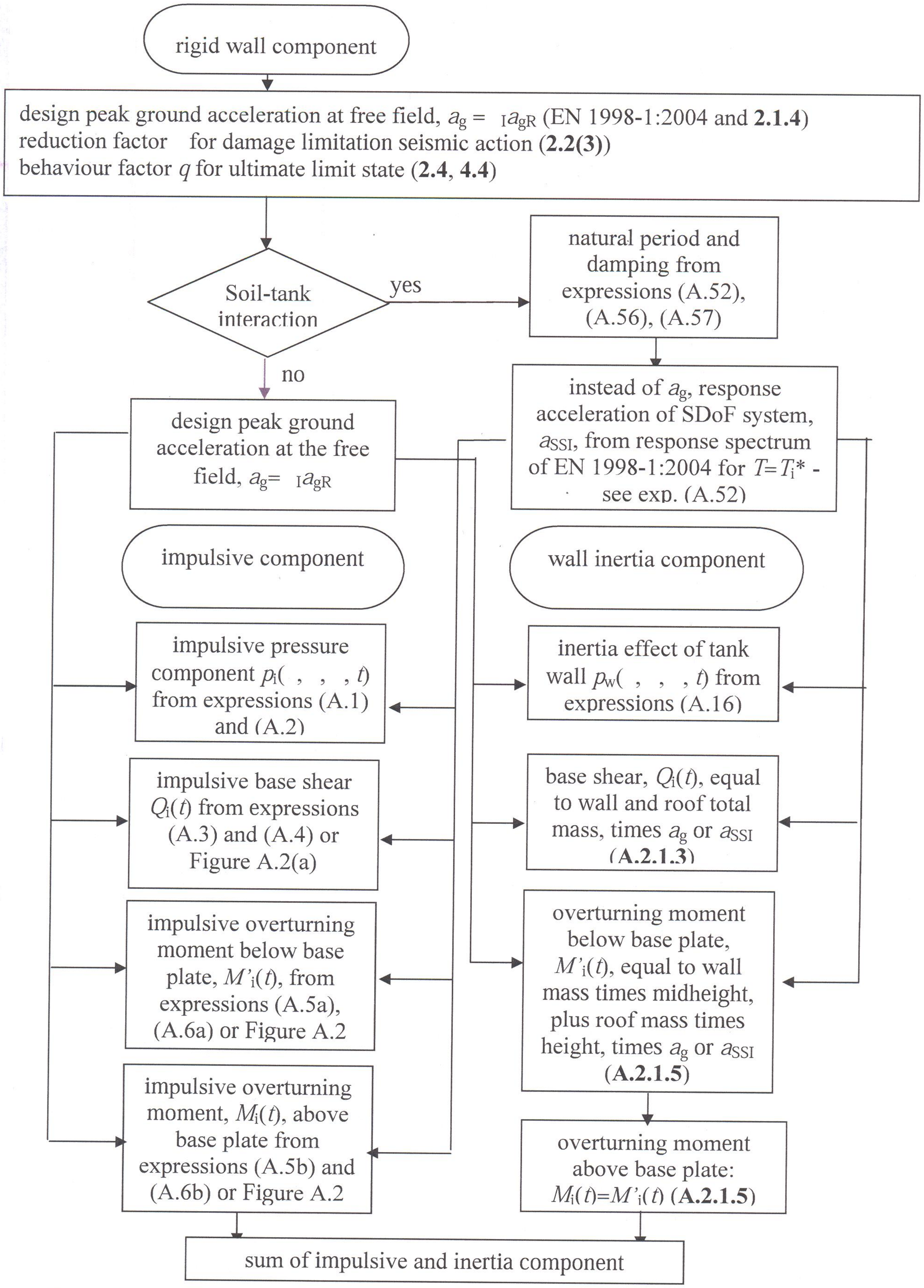
## A.8 Flow charts for calculation of hydrodynamic effects in vertical cylindrical tanks

The following flow charts provide an overview of the determination of hydrodynamic effects in vertical cylindrical tanks subjected to horizontal and vertical seismic actions. The flow charts essentially address the application of the response spectra method.

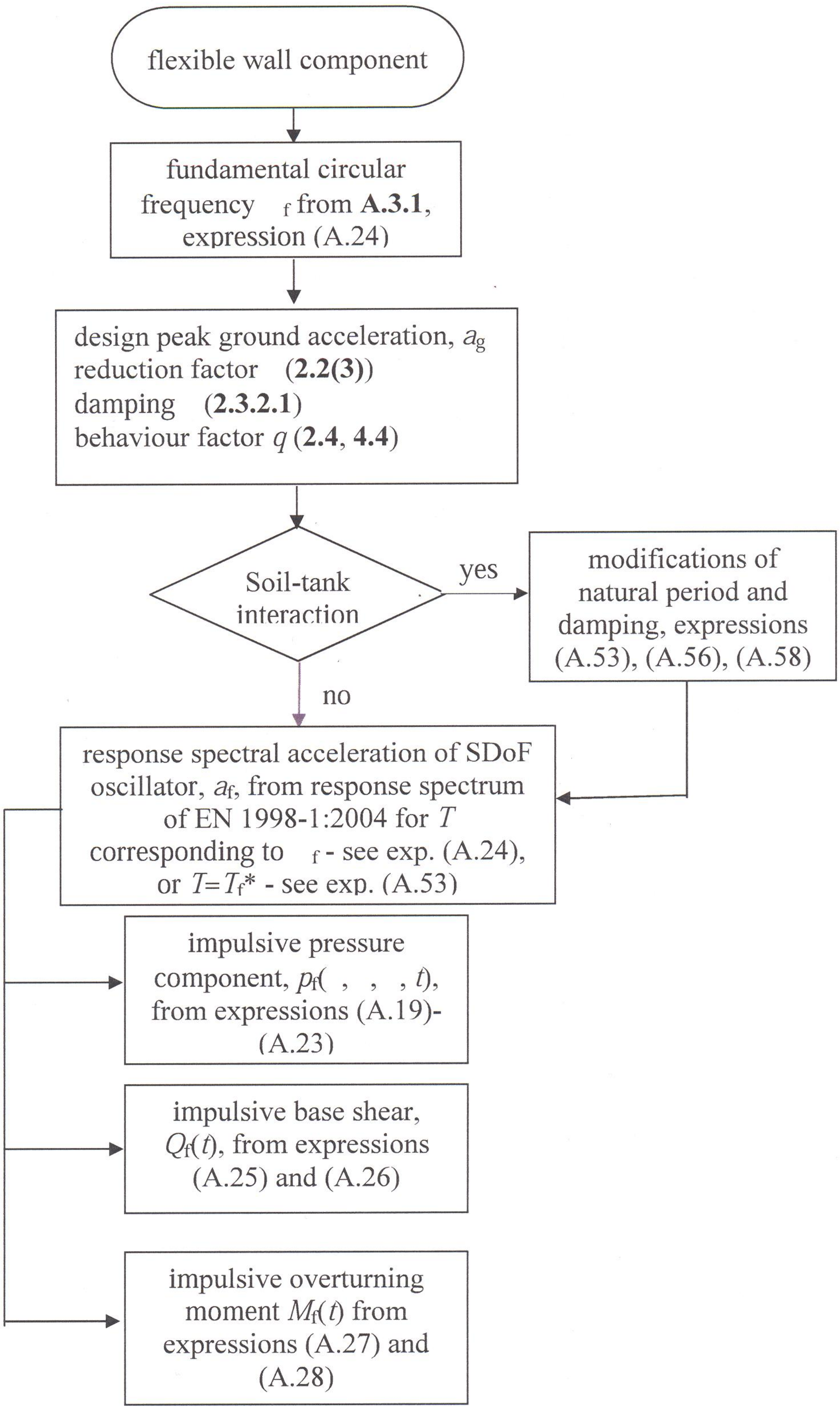
**Flow chart 1** gives an overview of the calculation process and of the combination of the various components of the response. **Flow charts 2** to **6** address the different hydrodynamic components or seismic action components.



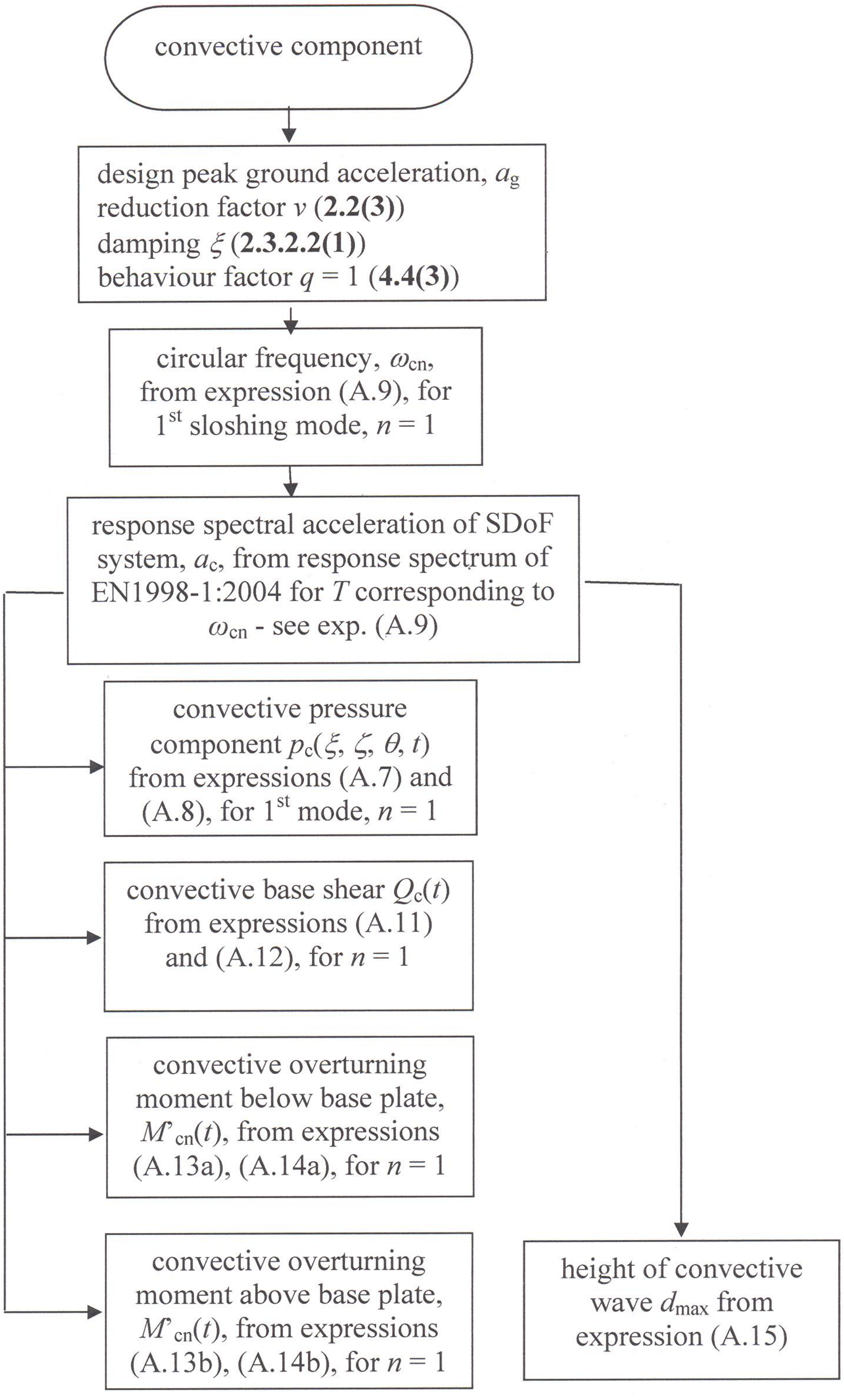

**Flow chart A.1: Overview of determination of hydrodynamic effects in anchored vertical cylindrical tanks on ground, considering soil-structure interaction**



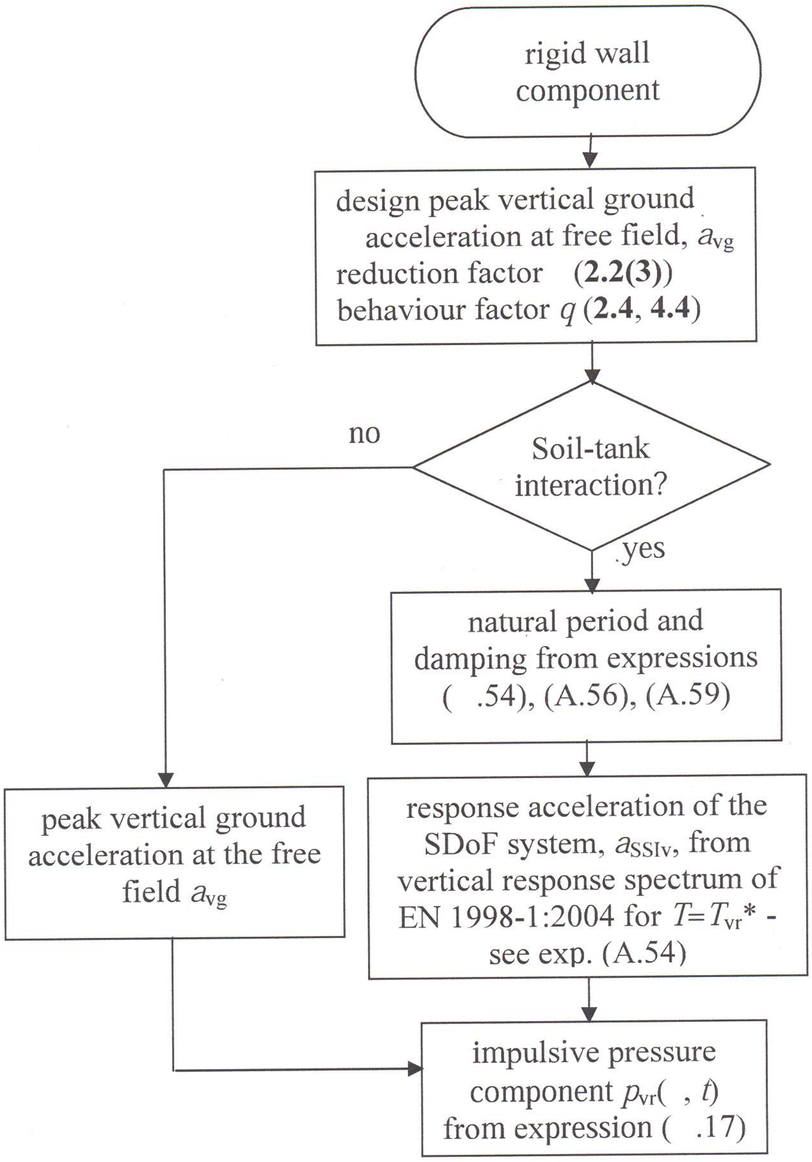

**Flow chart A.2: Horizontal seismic action, rigid wall impulsive component (see A.2.1, A.7.2)**



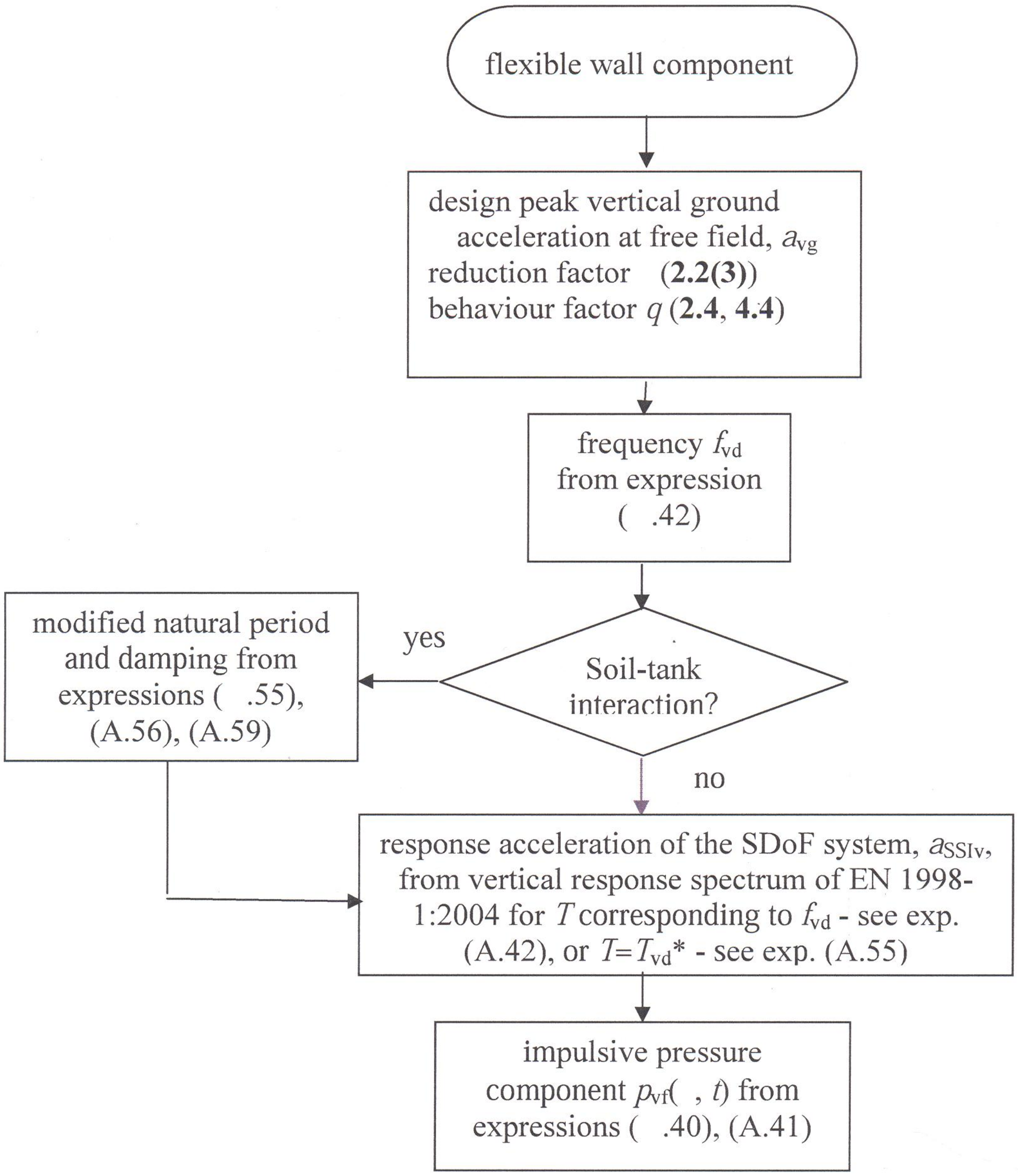

**Flow chart A.3: Horizontal seismic action, flexible wall impulsive component (see A.3.1, A.7.2)**




**Flow chart A.4: Horizontal seismic action, convective component (see A.2.1)**




**Flow chart A.5: Vertical seismic action, rigid wall component (see A.2.2, A.7.2)**




**Flow chart A.6: Vertical seismic action, flexible wall component (see A.3.3, A.7.2)**