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DYNAMIC RESPONSE OF RIGID STRIP-FOUNDATIONS BY A TIME-DOMAIN BOUNDARY ELEMENT METHOD

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SUMMARY

The dynamic response of rigid strip-foundations placed on or embedded in a homogeneous, isotropic, linear elastic half-space under conditions of plane strain to either external forces or obliquely incident seismic waves of arbitrary time variation is numerically obtained. The above mixed boundary-value problems are treated by the time domain boundary element method which is used in a step-by-step timewise fashion to provide the foundation response to a rectangular impulse. Numerical examples are presented in detail to demonstrate the use and importance of the proposed method. The method appears to be more advantageous than frequency domain techniques, because it provides the transient foundation response in a natural and direct way and can form the basis for extension to the non-linear case.

INTRODUCTION

In recent years, the need for the design of dynamically loaded structures such as machine foundations under external forces or various building foundations under seismic waves has spurred extensive research in the area of soil–structure interaction. A detailed review of research related to the dynamic response of rigid footings done until 1967 can be found in the book of Richard, Hall and Woods.[1] A recent state-of-the-art paper by Gazetas[2] covers the research activities in the area of foundation dynamics up to the year 1982, and a comprehensive review of the research pertinent to the dynamic analysis of three-dimensional foundations has been presented by Karabalis and Beskos.[3]

In this paper, the response of a two-dimensional rigid surface or embedded foundation to either external forces or oblique seismic waves is determined through a time domain BEM. The following discussion is restricted to two-dimensional rigid surface or embedded foundations placed on a linear elastic half-space. The feasibility of modelling a three-dimensional soil–structure system by a plane strain model, and the errors involved in such a representation have been explored by Luco and Hadjian[4] and Hadjian, Luco and Wong.[5]

The first successful analytical/numerical treatments of the dynamic behaviour of two-dimensional rigid surface foundations subjected to harmonic external forces can be found in References 6 and 7. Flitman[8] and Oien[9] considered the case of a rigid strip-foundation excited by

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harmonic Rayleigh seismic waves. Simpson,[10] following Oien's procedure, determined the response of a flexible one degree-of-freedom structure supported by a rigid foundation to Rayleigh wave excitation. The harmonic responses of rigid embedded foundations in an elastic half-space have also been studied analytically/numerically by Luco,[11] Trifunac,[12] Wong and Trifunac,[13] Luco, Wong and Trifunac[14] and Abdel-Ghaffar and Trifunac[15] who obtained the antiplane harmonic response of semicircular and semielliptical foundations to external forces and oblique SH waves. Thau,[16] Thau and Umek,[17,18] Umek[19] and Dravinski and Thau[20,21] also analytically/numerically examined both the antiplane and the general plane strain case of rectangular foundations subjected to either external forces or oblique seismic waves. Recently Meade and Keer[22] considered again the harmonic dynamic behaviour of a rigid line inclusion and a rectangular foundation subjected to antiplane harmonic shear waves.

The finite element method (FEM) in its conventional[23–25] and specialized forms[26,27] has also been used for the determination of the dynamic response of two-dimensional surface or embedded structures. Furthermore, simplified methods of analysis employing equivalent springs and dashpots to model the soil stiffness have been developed to approximately treat surface or embedded foundations.[28–31]

A more accurate and efficient numerical technique than the FEM for treating linear dynamic soil–structure interaction problems is the boundary element method (BEM).[32] Employing the BEM Dominguez[33,34] and Dominguez and Alarcon[35,36] have considered the time harmonic behaviour of two- and three-dimensional rigid surface or embedded foundations to external forces and travelling waves.

With the exception of the approximate method of springs and dashpots,[29–31] all the aforementioned methods used for studying the dynamics of rigid or flexible foundations[6–27,33–36] are frequency dependent methods providing only the harmonic foundation response. Use of the BEM permits one to study soil–structure interaction problems involving half-space modelling in the frequency as well as in the time domain. The time domain approach is more advantageous than frequency domain approaches since it provides directly the response in a natural way and forms the basis for extension to non-linear problems.[3,37,38]

A general time domain BEM has been developed for the dynamic analysis of soil–structure interaction problems and successfully applied by Spyrakos, Karabalis and Beskos,[37] Karabalis and Beskos[3] and Karabalis, Spyrakos and Beskos[38] to various surface three- and two-dimensional foundation problems. References 37 and 38 in connection with the two-dimensional case have been restricted to a brief treatment of surface foundations only. In the present paper, the general two-dimensional time domain boundary element methodology is developed and applied to surface as well as embedded dynamic strip-foundation problems. In addition, the present formulation develops the surface foundation analysis as a special case of the embedded foundation one, and Rayleigh waves are considered here instead of the P-waves considered in Reference 37 for comparison purposes with other known results. The present time domain BEM determines the response by superimposing the various impulse responses corresponding to the sequence of impulses representing the dynamic disturbance. This constitutes the primary difference from the time domain BEM formulations proposed by Cole, Kosloff and Minster,[39] Niwa *et al.*[40] and Manolis,[41] and allows one easily to circumvent difficulties encountered in the numerical treatment of the two-dimensional fundamental solutions.[42] In addition, instability and inaccuracy related to the accumulation of the error for late times[41] have not been encountered in the present formulation. An extension of the present approach to a direct step-by-step response determination in the time domain is straightforward and, in fact, has been presented by Karabalis and Beskos.[3] The response superposition approach was exclusively used here because of its accuracy and computational efficiency.[42]

INTEGRAL REPRESENTATION OF PLANE ELASTODYNAMICS

Under the assumptions of small displacement theory and homogeneous, isotropic, linear elastic material behaviour, the elastodynamic displacement field $u_i(\mathbf{x}, t)$ of a body D with a surface S under conditions of plane strain is governed by Navier's equation

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ii} + \rho f_j = \ddot{u}_j, \quad i, j = 1, 2 \tag{1}$$

where $f_j(\mathbf{x}, t)$ are the components of the body force per unit mass, λ and μ are the Lamé constants and ρ is the density of the medium.

The fundamental solution of Navier's equation, the response of an infinite medium to a unit impulse body force uniformly distributed along the line perpendicular to the plane D at point ξ and acting at time τ, is given by[43]

$$\begin{aligned} v_{ij}(\mathbf{x}, t; \xi \mid f) = \frac{1}{2\pi\rho} \Bigg\{ \frac{\partial^2}{\partial \xi_i \partial \xi_j} \Bigg[& H\left(t - \frac{r}{c_1}\right) \int_r^{c_1 t} \frac{\mathrm{d}\eta}{(n^2 - r^2)^{1/2}} \\ & \times \int_0^{t-(\eta/c_1)} v f\left(t - \frac{\eta}{c_1} - v\right) \mathrm{d}v - H\left(t - \frac{r}{c_2}\right) \int_r^{c_2 t} \frac{\mathrm{d}\eta}{(n^2 - r^2)^{1/2}} \\ & \times \int_0^{t-(\eta/c_2)} v f\left(t - \frac{\eta}{c_2} - v\right) \mathrm{d}v \Bigg] + \frac{\delta_{ij}}{c_2^2} H\left(t - \frac{r}{c_2}\right) \int_r^{c_2 t} f\left(t - \frac{\eta}{c_2}\right) \frac{\mathrm{d}\eta}{(n^2 - r^2)^{1/2}} \Bigg\} \end{aligned} \tag{2}$$

$$r_i = x_i - \xi_i, r^2 = (x_i - \xi_i)(x_i - \xi_i), \quad \eta^2 = r^2 + x_3^2; \quad i = 1, 2 \tag{3}$$

where x_3 is the axis perpendicular to the plane of the body D, and H represents the Heaviside function.

In a well-posed boundary value problem, the equations of motion (1) can be reduced to an integral equation form.[43,44] This is accomplished through the elastodynamic reciprocal theorem relating the elastodynamic state to be determined with the fundamental elastodynamic state expressed by equation (2). Using the dynamic reciprocal theorem and under the assumption of zero body forces and initial conditions, one can derive the following integral identity:[43,45]

$$\varepsilon(\xi) u_j(\xi, t) = \int_S \{ v_{ij}[\mathbf{x}, t; \xi \mid t_{(\mathbf{n})i}(\mathbf{x}, t)] - \sigma_{(\mathbf{n})ij}[\mathbf{x}, t; \xi \mid u_i(\mathbf{x}, t)] \} \, \mathrm{d}S(\mathbf{x}) \tag{4}$$

where the stress tensor $\sigma_{(\mathbf{n})ij}$ is related to the fundamental solution v_{ij} through the constitutive relations defined by Eringen and Suhubi.[43] For a boundary smooth at ξ ($\xi \in S$) $\varepsilon(\xi) = \delta_{ij}/2$; for the interior ($\xi \in D$) $\varepsilon(\xi) = \delta_{ij}$, and for the exterior ($\xi \notin D$) $\varepsilon(\xi) = 0$.

Although the above integral representation provides a very elegant expression of the solution to any transient elastodynamic problem, an analytical solution of such a boundary value problem is impossible for the general case. Thus, resort is usually made to numerical methods of solution. This numerical treatment of equation (4) is presented in the next section.

NUMERICAL TREATMENT OF INTEGRAL EQUATIONS

In soil–structure interaction problems, where the primary purpose is the determination of the contact stresses at the soil–structure interface, one has to combine through equilibrium and compatibility the behaviour of the two components, soil and structure, expressed by their pertinent governing equation. The behaviour of the half-plane soil medium can be expressed by an integral equation of the form of equation (4).

A numerical treatment of the boundary integral equation (4) with $\varepsilon(\xi) = 1/2$ involving both time

and spatial discretization is presented in this section. The time discretization assumes an approximation of the externally applied surface tractions as a sequence of N rectangular impulses of equal duration Δt, as shown in Figure 1(a). The spatial discretization requires division of the boundary S of the half-plane soil medium into Q line elements of length DS_g. Figure 1(b) shows such a spatial discretization for the case of a surface strip-foundation.

The essential steps in the solution process are (a) evaluation of the boundary response u_j due to a single rectangular impulse traction $t_{(\mathbf{n})i}$, (b) superposition of the individual impulse responses to obtain the total response.

On account of the causality property[43] of the fundamental solution $v_{ij}(\mathbf{x}, t; \boldsymbol{\xi} \mid t_{(\mathbf{n}),i})$ only the first time step ($n = 1$) numerical treatment of this solution is required. Thus, using the spatial and time discretizations, equation (2) leads to

$$v_{ij}(\mathbf{x}, t; \boldsymbol{\xi} \mid t_{(\mathbf{n})j}) = \frac{1}{2\pi\rho}\left\{\frac{\partial^2}{\partial\xi_i\partial\xi_j}\left[H\left(t-\frac{r}{c_1}\right)\int_r^{c_1 t}\frac{F_1\,\mathrm{d}\eta}{(\eta^2-r^2)^{1/2}} - H\left(t-\frac{r}{c_2}\right)\right.\right.$$
$$\left.\left.\times\int_r^{c_2 t}\frac{F_2\,\mathrm{d}\eta}{(\eta^2-r^2)^{1/2}}\right]t_1(\mathbf{n}) + \frac{\delta_{ij}}{c_2^2}H\left(t-\frac{r}{c_2}\right)\int_r^{c_2 t}\frac{t_{(\eta)i}\left(\eta, t-\dfrac{\eta}{c_2}\right)}{(\eta^2-r^2)^{1/2}}\,\mathrm{d}\eta\right\} \tag{5}$$

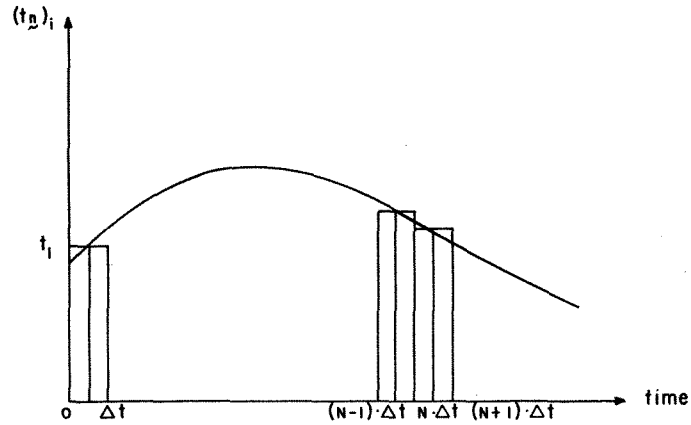

Figure 1(a). Discretization of the contact traction $t_{(\mathbf{n})i}$ to a sequence of rectangular impulses

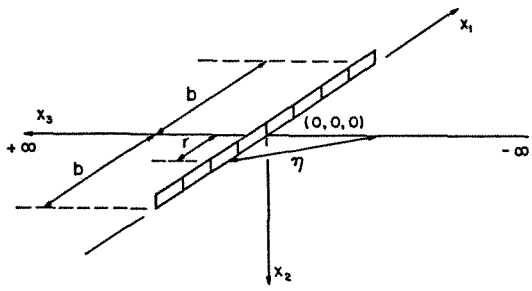

Figure 1(b). Geometry and discretization of a surface strip-foundation

where

$$F_a = \begin{cases} \frac{1}{2}\left(t - \frac{\eta}{c_\alpha}\right)^2, & \text{if } 0 < t - \frac{\eta}{c_\alpha} < \Delta t \\ \left(t - \frac{\eta}{c_\alpha}\right)\Delta t - \frac{1}{2}(\Delta t)^2, & \text{if } \Delta t < t - \frac{\eta}{c_\alpha}, \alpha = 1, 2 \end{cases} \tag{6}$$

with c_1 and c_2 being the dilatational (P) and shear (S) wave velocities, respectively, $t_1(\mathbf{n})$ is the intensity of the traction vector and η is shown in Figure 1(b) and given by equation (3). The indicated spatial integrations and differentiations are performed analytically, leading to the discretized form G_{ij}^{nq}. Similar treatment of the tensor $\sigma_{(\mathbf{n})ij}$ yields the discretized kernel function F_{ij}^{nq}. Both G_{ij}^{nq} and F_{ij}^{nq} are given explicitly in Reference 42.

On account of the discretized kernel function forms G_{ij}^{nq} and F_{ij}^{nq} and on the assumption of constant variation of displacements and tractions over each line element and time step n, equation (4), written for every boundary element p, yields the following system of linear algebraic equations:

$$\tfrac{1}{2}u_p^{Np} = \sum_{n=1}^{N}\sum_{q=1}^{Q}\left(\left[\int_{\Delta s} G^{nq}\,\mathrm{d}s\right]\{t^{lq}\} - \left[\int_{\Delta s} F^{nq}\,\mathrm{d}s\right]\{u^{lq}\}\right), \tag{7}$$

where $n = 1, 2, \ldots, N$, $l = N + n - 1$, and $q = 1, 2, \ldots, Q$.

The $\int_{\Delta s} G^{nq}\,\mathrm{d}s$ and $\int_{\Delta s} F^{nq}\,\mathrm{d}s$ physically represent the displacements and tractions respectively, at the centre of an element p at the time step n, which are induced by a unit rectangular impulse of the constant traction vector acting over the element q during the first time step.

Finally, the total system response to a sequence of impulses starting at time $m\Delta t$, i.e. to the time history of the applied traction $(N - m)\Delta t$, as shown in Figure 2, can be evaluated from

$$\tfrac{1}{2}u^{Np} = \sum_{q=1}^{Q}\sum_{n=m}^{N}\left(\left[\int_{\Delta s} G^{lq}\,\mathrm{d}s\right]\{t^{N-l+1}\} - \left[\int_{\Delta s} F^{lq}\,\mathrm{d}s\right]\{u^{N-l+1}\}\right) \tag{8}$$

where $l = n - m + 1$.

The integrals of equation (8) present logarithmic singularities which appear only when the 'source element' p and the 'receiver element' q coincide. The treatment of the

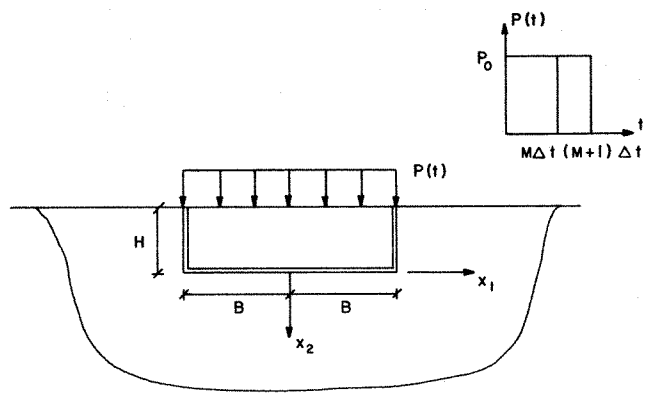

Figure 2. Geometry of an embedded rigid strip-foundation to external forces

logarithmic singularities is presented in detail in Reference 42. Except for these singularities, the integrations indicated in equations (7) and (8) present no difficulties, and are performed numerically using a Gaussian quadrature algorithm.

EXTERNALLY APPLIED DYNAMIC LOADS

Consider an embedded rigid massless foundation with a rectangular cross-section and with infinite length along the x_3 axis, as shown in Figure 2. The foundation is placed on a homogeneous, isotropic, linear elastic soil and subjected to a rectangular impulse force at time step m. Since the fundamental solution of the infinite plane is employed, the boundary to be discretized will be both the contact area and the free surface. However, a relatively small area outside the soil–foundation interface needs to be discretized, because the influence of the free field elements on the foundation response is decreasing with distance.[35] For a discretization of the soil–foundation interface and part of the adjacent free field into a number, Q, of elements, as shown in Figure 3(a), equation (8) can be expressed in the following matrix form:

$$\tfrac{1}{2}\{u^N\} = \left[\int_s G^m \, \mathrm{d}s\right]\{t^N\} + \{\mathrm{PG}^N\} - \{\mathrm{PF}^N\} \tag{9}$$

where $\{u^N\}$ and $\{t^N\}$ are the displacement and traction vectors at time $N\Delta t$ due to an impulse force at time $m\Delta t$, respectively. The components of the vectors $\{\mathrm{PG}^N\}$ and $\{\mathrm{PF}^N\}$ are given by

$$\{\mathrm{PG}^N\} = \delta_{Nm} \sum_{n=m+1}^{N} \left[\int_s G^l \, \mathrm{d}s\right]\{t^{N-l+1}\}$$

and

$$\{\mathrm{PF}^N\} = \delta_{Nm} \sum_{n=m+1}^{N} \left[\int_s F^l \, \mathrm{d}s\right]\{u^{N-l+1}\} \tag{10}$$

respectively, where δ_{Nm} is the Kronecker's delta $\{\mathrm{PG}^N\} - \{\mathrm{PF}^N\}$ can be interpreted as the influence of all the previous time steps ($n < N$) on the foundation response.

The compatibility of displacements at the soil–foundation interface is given by

$$\{u_c^N\} = [S]\{D^N\} \tag{11}$$

where $\{u_c^N\}$ is the vector of the displacements at the centre of the rigid foundation elements along the x_1 and x_2 axes, and $\{D^N\}$ is the vector of the vertical (Δ_2^N), horizontal (Δ_1^N) and rotational (ϕ_3^N) response amplitudes. The entries of the matrix $[S]$ are functions of the geometric quantities[42] R_0 and θ_1 shown in Figure 3(a).

At a time step m the externally applied impulse force $\{P^m\}$ can be expressed in terms of the

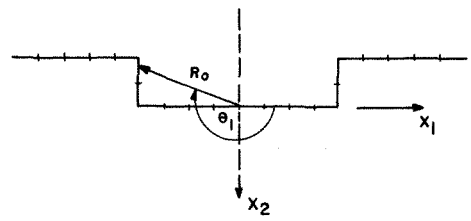

Figure 3(a). Spatial discretization of an embedded rigid strip-foundation

contact stresses as

$$\{P^m\} = [K]\{t^m\} \tag{12}$$

where the entries of the matrix $[K]$ are functions of the geometric quantities of the foundation.[42]

Equations (9), (11) and (12) form a system of linear algebraic equations which can be solved for the unknown contact displacements and tractions developed at the contact area of an embedded foundation in complete bond with the elastic half-space. However, there is an insignificant effect on solution accuracy for most practical purposes[35] and the computation effort is greatly reduced, if 'rigid-smooth' or 'rigid-pressureless' conditions[18] (i.e. relaxed boundary conditions) are adopted at the soil–foundation interface. In Reference 42, solutions of the above system of linear algebraic equations are given explicitly in matrix form.

The surface rigid foundation can be considered as the limiting case of the associated embedded foundation with an embedment approaching the zero value. For the case of rigid surface strip-foundations, however, the effect on the response of the elements outside the interface is very small, and a solution of acceptable accuracy can be obtained without any free field elements.[34,35] In fact, the response and tractions at the contact area of the surface foundation in complete bond with the elastic half-space can be determined from equations (9), (11) and (12), which for $H = 0$ can be solved to give

$$\{D^N\} = \left([K]\left[\int_s G^m \,\mathrm{d}s\right]^{-1}[S]\right)^{-1}\left(\{P^m\} + [K]\left[\int_s G^m \,\mathrm{d}s\right]^{-1}\{\mathrm{PG}^N\}\right) \tag{13}$$

and

$$\{t^N\} = \left[\int_s G^m \,\mathrm{d}s\right]^{-1}([S]\{D^N\} - \{\mathrm{PG}^N\}) \tag{14}$$

In order to determine the response of a rigid embedded or surface footing to a sequence of rectangular impulses approximating an external transient load, as shown in Figure 1(a), the individual impulse responses given by either equation (13) or its counterpart for the embedded foundation[42] are superimposed as described by equation (8).

OBLIQUELY INCIDENT SEISMIC WAVES

Consider an embedded rigid massless strip-foundation in a homogeneous linear elastic medium representing the soil and subjected to plane seismic waves impinging on its bottom side at an angle θ, as shown in Figure 3(b). On the free surface, the total displacement field can be expressed as[46,47]

$$\{u_f(x_1, x_2, t)\} = [u_{f1}(x_1, x_2, t)\ \ u_{f2}(x_1, x_2, t)]^T \tag{15}$$

where u_{f1} and u_{f2} are the displacement components about the axes x_1 and x_2, respectively. The presence of a rigid footing embedded in the elastic medium causes scattering of the waves impinging on the foundation. The total response of the foundation, resulting from superposition of the free and scattered fields, can be written in the form[48]

$$\{u(x_1, t)\} = \{u_f(x_1, t)\} + \{u_s(x_1, t)\} \tag{16}$$

Approximating the time variation of the continuous free and scattered fields by a sequence of rectangular impulses and discretizing the contact area into Q_1 equal elements, the response of a massless strip-footing at a time step N due to a rectangular impulse free-field displacement acting

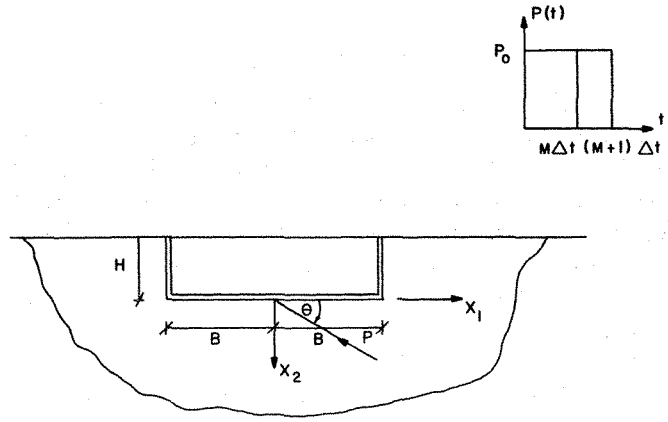

Figure 3(b). Geometry of an embedded rigid strip-foundation to seismic waves

at a time step m can be decomposed as

$$\{u^N\} = \{u_f^N\} + \{u_s^N\}, \quad \text{for} \quad N = m$$

and

$$\{u^N\} = \{u_s^N\} = \{0\}, \quad \text{for} \quad N > m \tag{17}$$

where the vectors $\{u^N\}$, $\{u_f^N\}$ and $\{u_s^N\}$ represent the total, free and scattered displacements at the centre of each element, respectively.

Under the assumption of relaxed boundary conditions, the decoupled scattered field displacements are related to the total tractions simply by

$$\tfrac{1}{2}\{u_s^N\} = \left[\int_s G^N \, ds\right]\{t^N\} \tag{18}$$

In view of the above equation, the vector of the externally applied forces given by equation (12) can be expressed in terms of the scattered field displacements. The resulting expression, combined with equations (11) and (17), forms a system of linear algebraic equations which can be solved for the displacement vector[42] $\{D^N\}$.

The case of surface foundations can be deduced from the embedded case for zero embedment. In fact, on the lines of the above developments, the response of a rigid massless surface foundation in complete bond with the elastic half-space is given by[42]

$$\{D^N\} = \left([K]\left[\int_s G^N \, ds\right]^{-1}[S]\right)^{-1}[K]\left[\int_s G^N \, ds\right]^{-1}\{u_f^N\} \tag{19}$$

NUMERICAL EXAMPLES

Two numerical examples are presented in order to demonstrate the applicability and the accuracy of the time domain BEM. The method is employed to determine the responses of a surface and an embedded massless rigid strip-foundation subjected to either external dynamic forces or seismic waves. Even though the responses of the foundations to a transient dynamic disturbance can be obtained from the rectangular impulse responses in conjunction with the superposition algorithm expressed by equation (8), the steady state responses of the footings are obtained here by the

proposed method in order to compare the accuracy of the method against already existing results in the frequency domain.[6,7,9,10,35]

Example 1

Consider a surface rigid strip-foundation with 5·0 ft width, placed on the surface of a homogeneous isotropic linear elastic half-space characterized by a modulus of elasticity $E = 2{\cdot}58984 \times 10^9\,\mathrm{lb/ft^2}$, a mass density $\rho = 10{\cdot}368\,\mathrm{lb/sec^2/ft^4}$ and Poisson's ratio $\nu = 1/3$ or $1/4$.

The foundation is first subjected to the rectangular impulse forces and moment with intensities

$$P_{10} = P_{20} = 180\,\mathrm{k/ft}, \quad M_{30} = 180\,\mathrm{k\text{-}ft/ft} \tag{20}$$

respectively, and time duration $\Delta t = 16 \times 10^{-6}$ s for $\nu = 1/3$. Even though the present methodology can take into account non-relaxed boundary conditions (perfect bonding), 'rigid-smooth' or 'rigid-pressureless' conditions are assumed at the contact area permitting the decoupling of the vertical, horizontal and rocking motions. The assumption of relaxed boundary conditions reduces the computational effort considerably without any significant effect on the solution accuracy. Results are given for a discretization consisting of 8 equal line elements; however, essentially identical results have also been obtained for a 5-element mesh. In order to increase the accuracy of the method, as far as the dynamic effect of the travelling waves on the response is concerned, every element is further discretized into 5 subelements. Thus, each component of the tensor $[\int_{\Delta s} G^{nq}\,\mathrm{d}s]$ of equation (7) is obtained by adding all the values corresponding to each individual subelement. The vertical response of the footing is plotted versus time in Figure 4. The Figures depicting the horizontal and rocking impulse responses can be found in Reference 42. The accuracy of the

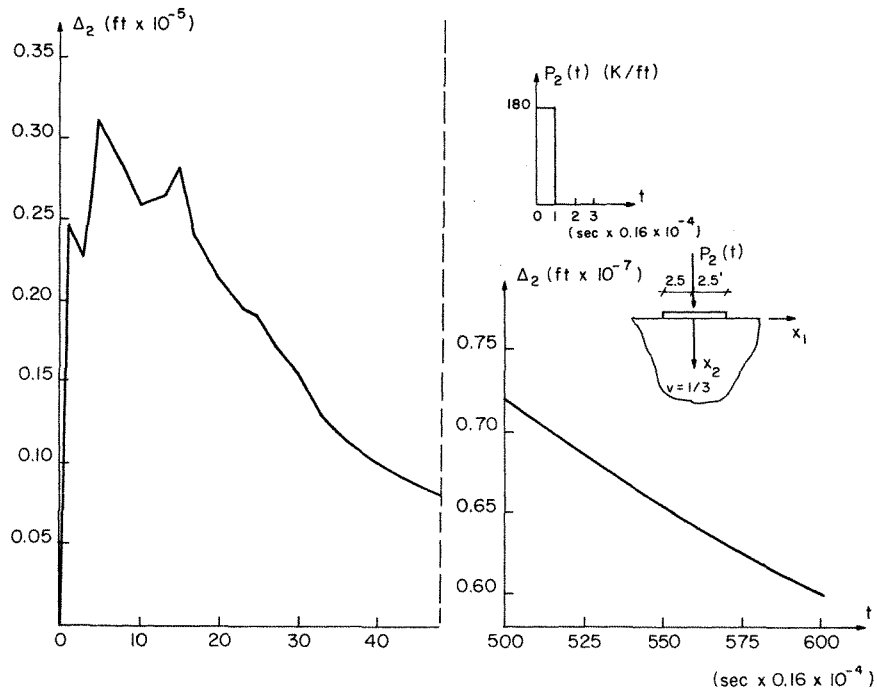

Figure 4. Vertical force impulse response

impulse responses has been verified through comparison studies presented by Spyrakos and Antes.[48]

In order to document the accuracy of the method, the response of the foundation to harmonic forces is determined. The external forces have the form

$$P_1(t) = P_{10} \sin \omega t, \quad P_2(t) = P_{20} \sin \omega t, \quad M_3(t) = M_{30} \sin \omega t \tag{21}$$

where $P_{10} = P_{20} = 180$ k/ft and $M_{30} = 180$ k–ft/ft. The vertical, horizontal and rocking harmonic amplitudes, as obtained by the proposed method and shown in Figures 5(a)–(c), are in close

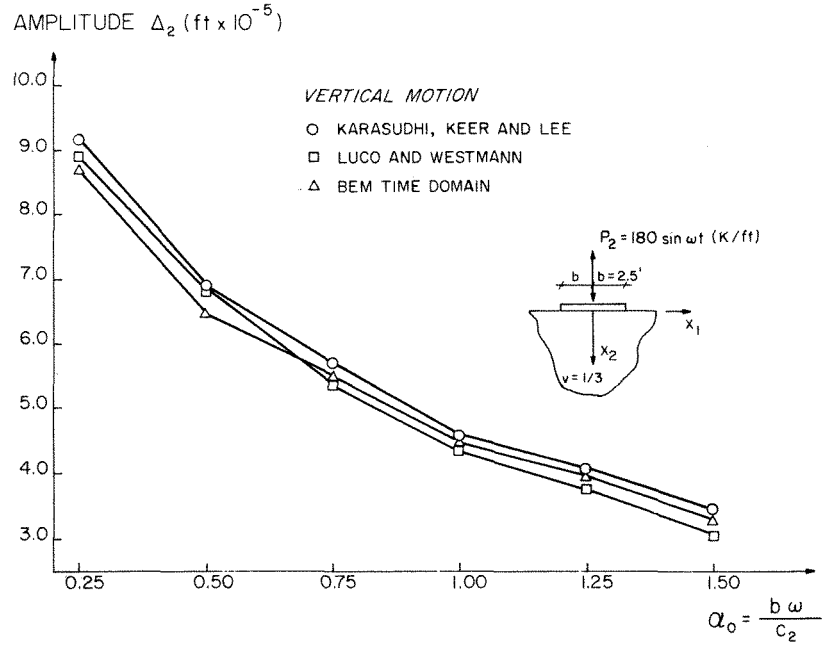

Figure 5(a). Vertical harmonic force response amplitude

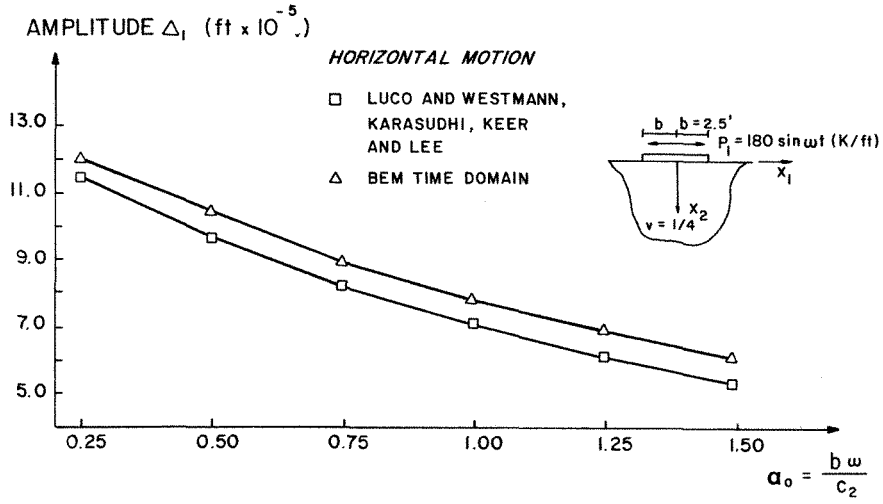

Figure 5(b). Horizontal harmonic force response amplitude

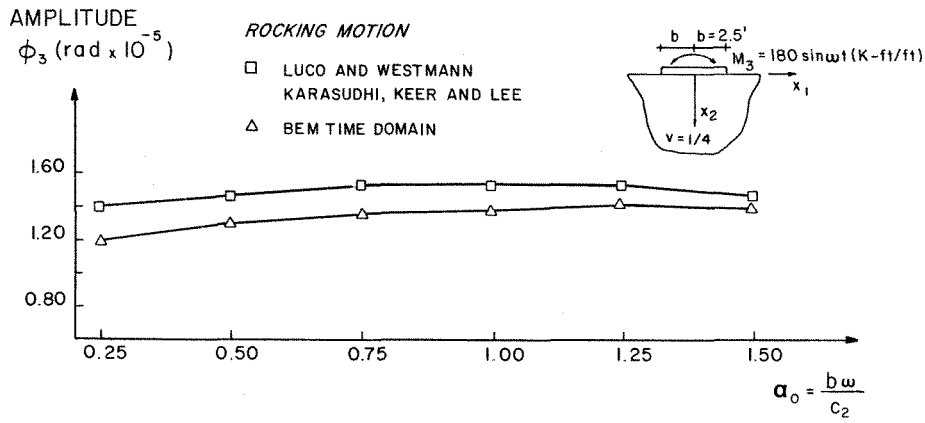

Figure 5(c). Rocking harmonic force response amplitude

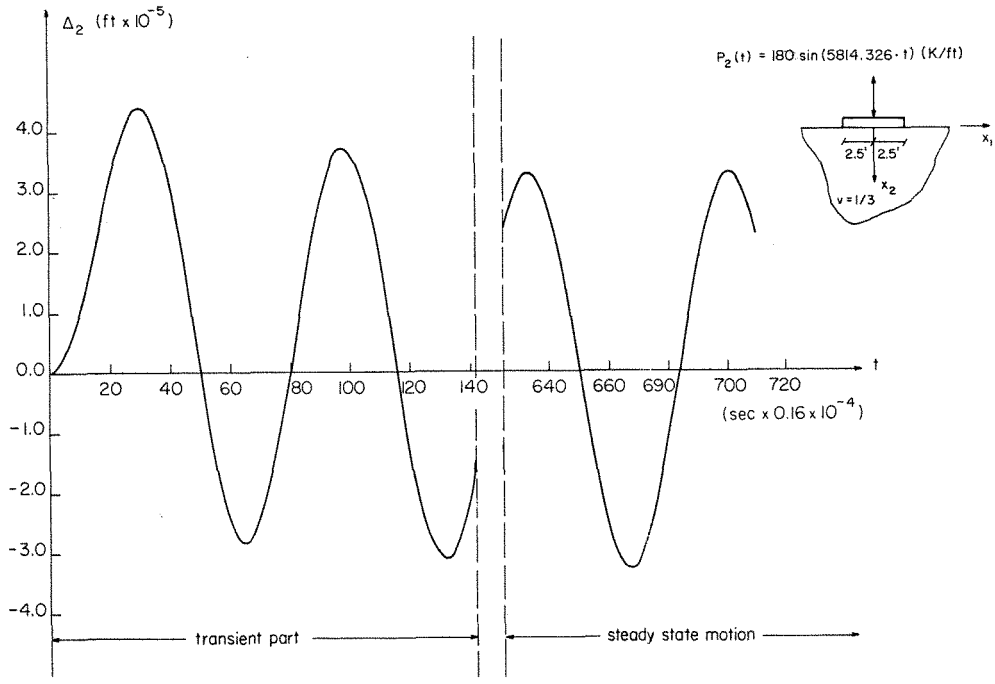

Figure 6. Vertical harmonic force response

agreement with the results presented by Karasudhi, Keer and Lee[6] and Luco and Westmann.[7] Figure 6 depicts the time history of the vertical harmonic response of the footing for a specific frequency. The remaining responses for the horizontal and rocking motions have been plotted in Reference 42. It should be pointed out, as it is clearly shown in Figure 6, that the present time domain BEM methodology, in contrast to the conventional frequency domain techniques, can capture the transient motion during the early response time. All required computations were done on the University of Minnesota CDC Cyber 74 computer. A typical response calculation (vertical impulse response) for 8 line elements with 5 subelements per element and 600 time steps required

650·694s CP time, whereas the corresponding impulse superposition for the computation of the vertical harmonic response only 3·048s CP time. The above mentioned CP times correspond to the determination of the responses plotted in Figure 4 and Figure 6, respectively. All the required numerical integrations were performed by a six-point Gaussian quadrature algorithm.

Consider now the same strip-foundation subjected to a train of plane travelling waves propagating in the x_1x_2 plane. The vector of the total displacements of the rigid massless footing for seismic waves of a general time variation can be determined from equation (19). Even though equation (19) permits the calculation of the foundation response to a general train of seismic waves, the response is obtained here for a harmonic Rayleigh wave in order to compare the accuracy of the method with already existing results.[9,10] In the absence of the rigid foundation and for an elastic soil medium with $\nu = 1/4$, the vertical free-field surface displacement u_{f2} with an amplitude R_v for a harmonic Rayleigh wave horizontally propagating in the x_1x_2 plane is given by[46]

$$u_{f2} = R_v \sin\left[\omega\left(t - \frac{x_1}{9190{\cdot}168}\right)\right] \tag{22}$$

The results obtained by the time domain BEM under relaxed boundary conditions show a very close agreement with the results of Oien[9] and Simpson[10] as indicated in Figures 7(a) and 7(b). It should be noticed that although Simpson's[10] results have been obtained under the assumption of complete bonding they are very close to the present results.

Even though a Harris H100, which is by far smaller and slower than a Cyber 74, was used to perform the required computations only 42·85s CP time was needed for the calculation of the seismic impulse response[42] and 52·59s CP time for 1800 steps of the seismic harmonic response. This very small amount of CP time was expected, since no superposition was employed for the determination of the seismic motions.

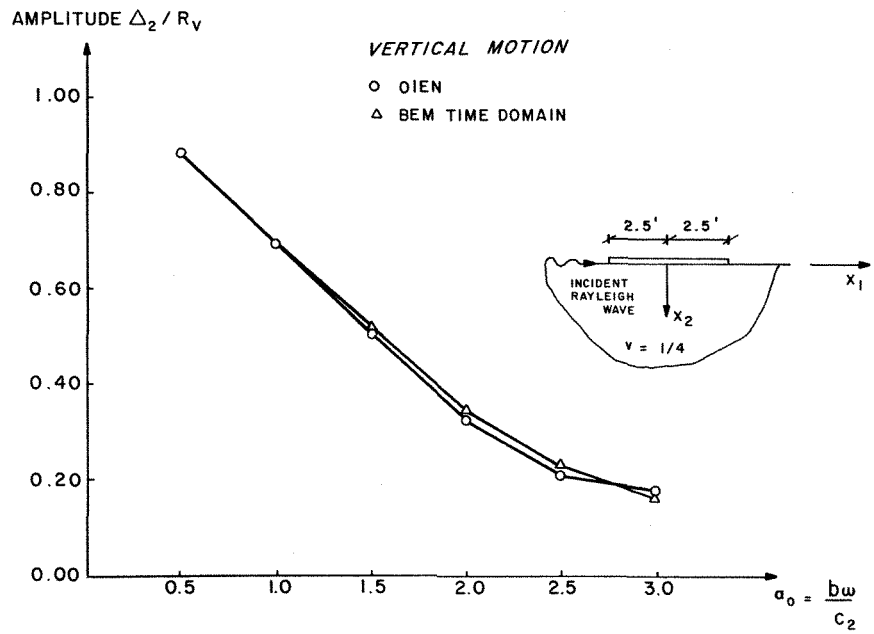

Figure 7(a). Vertical harmonic Rayleigh wave response amplitude

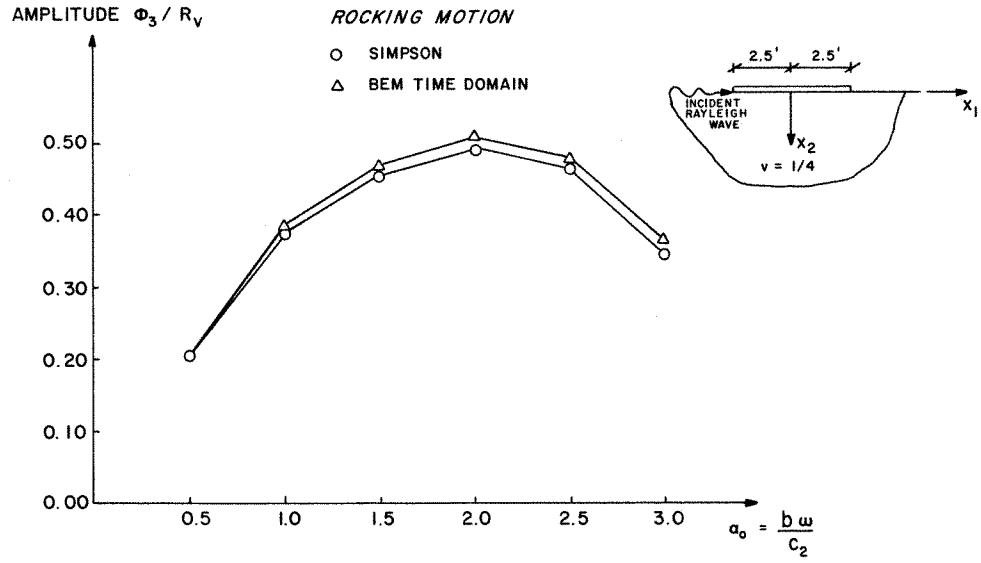

Figure 7(b). Rocking harmonic Rayleigh wave response amplitude

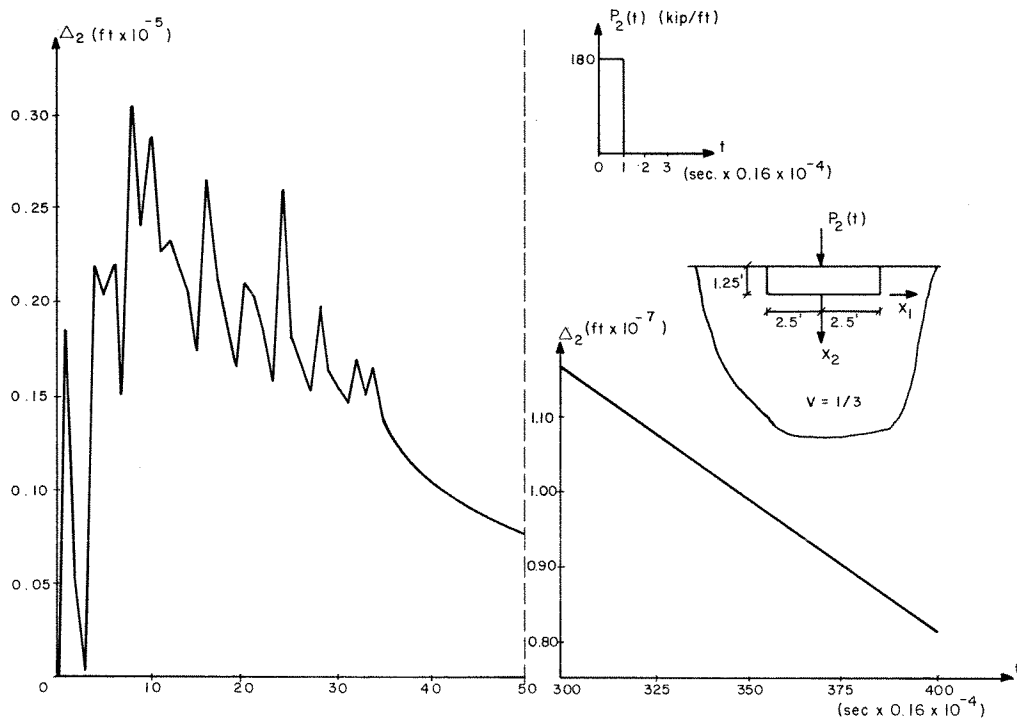

Figure 8. Vertical force impulse response

Example 2

Consider an embedded strip-foundation of 5·0 ft width and 1·25 ft depth placed on a linear elastic soil medium characterized by the elastic constants and density defined in the first example.

Assuming relaxed boundary conditions, the response of the embedded strip-footing to the rectangular impulse forces and moment of magnitude $P_{10} = P_{20} = 180$ k/ft and $M_{30} = 180$ k–ft/ft, respectively, and time duration $\Delta t = 16 \times 10^{-6}$ s is determined first. The soil–foundation interface and part of the soil around the foundation is discretized into 20 elements as shown in Figure 3(a). In addition, a subdivision of every element into 3 subelements has been employed in order to improve the computational accuracy of the tensors $[\int_s G^m \, ds]$ and $[\int_s F^l \, ds]$ appearing in equations (9) and (10) in an efficient way. The vertical impulse response of the foundation is plotted in Figure 8. It is observed that the radiation damping quickly smoothes out the irregularities of the impulse response occurring at early times, almost at the rate observed for surface footings. Similar Figures pertaining to the horizontal and rocking impulse responses are given in Reference 42. Owing to the additional resistance imposed by the side walls, the amplitudes of the impulse responses for the embedded foundation are smaller than their counterparts of the surface case. The results portrayed in the above Figures are sufficient to determine the response of the footing to any external dynamic loading.

In order to document the accuracy of the method, the response of the foundation to the harmonic forces expressed by equation (21) are determined next. The resulting amplitudes of the harmonic motions, together with those obtained by Alarcon and Dominguez,[35] are plotted in Figures 9(a)–(c).

It is observed that the vertical, horizontal and rocking amplitudes of the present analysis are in close agreement with those of reference 35. The small differences between the solutions of the two methods could be due to the fact that the results of reference 35 were reproduced here by enlarging

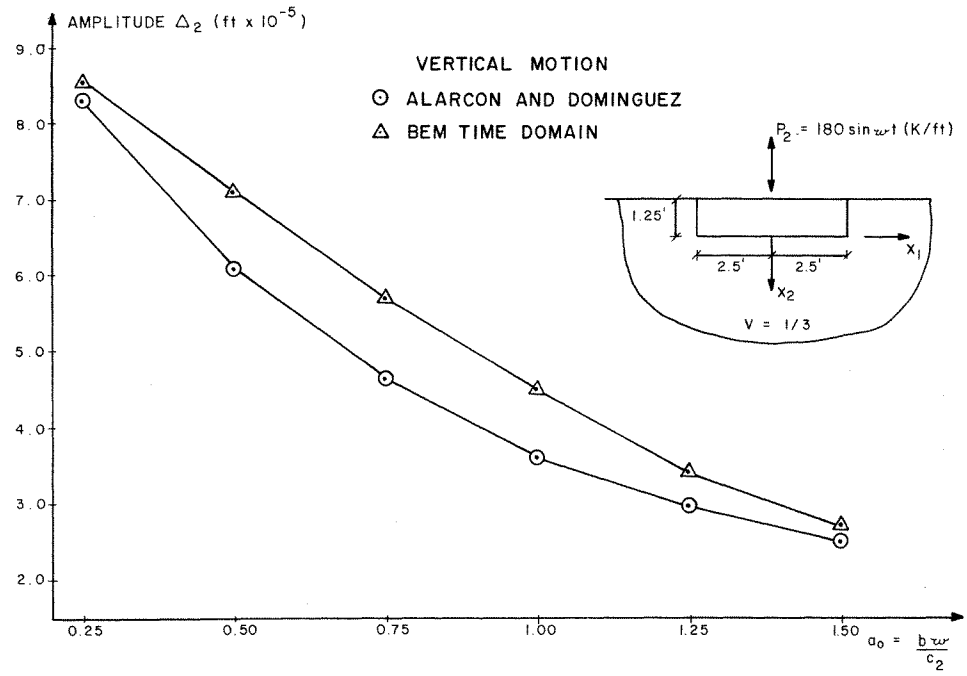

Figure 9(a). Vertical harmonic force response amplitude

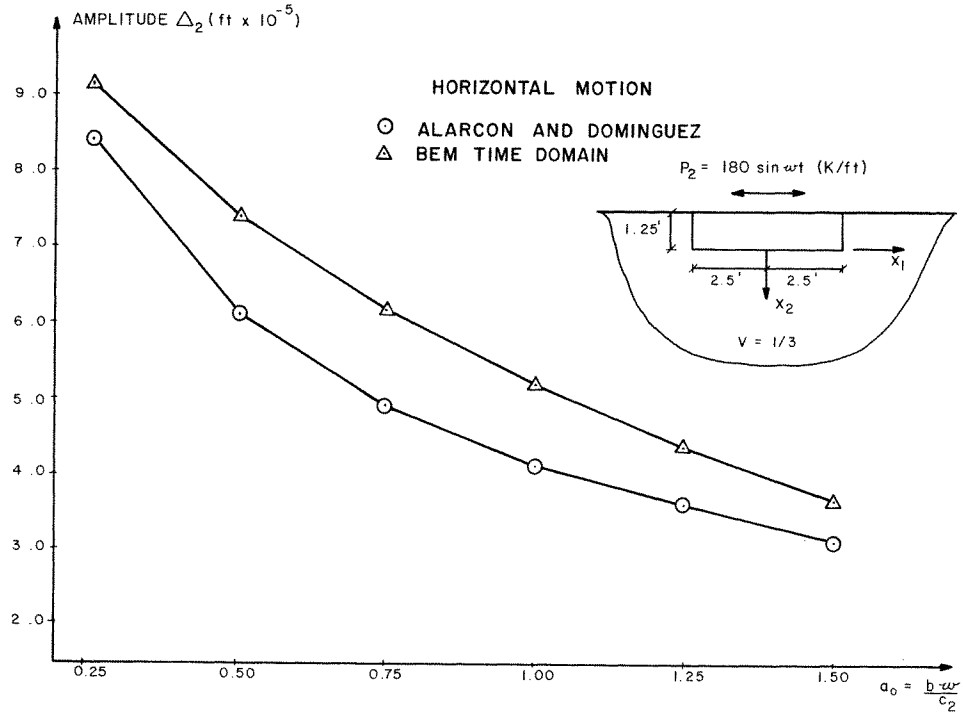

Figure 9(b). Horizontal harmonic force response amplitude

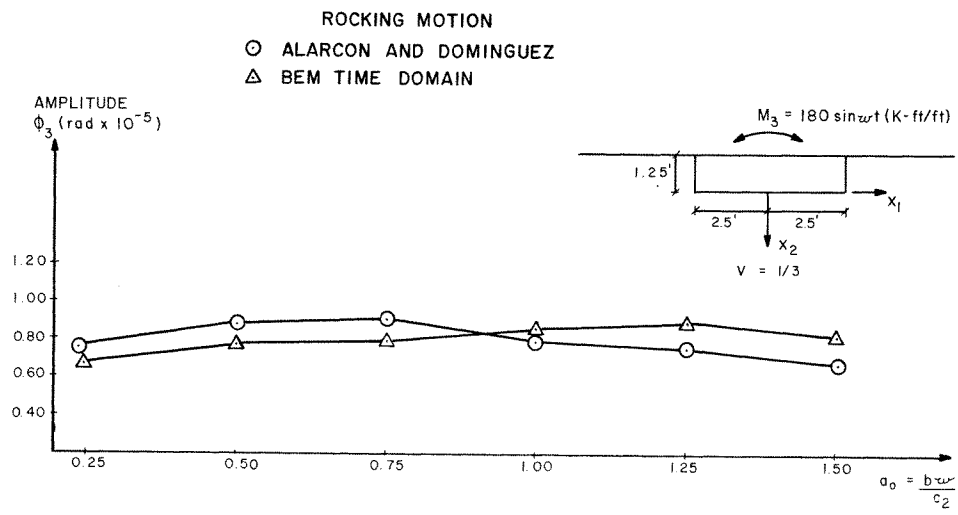

Figure 9(c). Rocking harmonic force response amplitude

their very small figures. The response of the footing to a vertical harmonic force is plotted in Fig. 10.

Let now the foundation be excited by plane compressional waves impinging on the bottom side at $\theta = 75°$ angle of incidence. Even though the foundation response to a general train of compressional waves can be evaluated with the aid of equation (19), the response is obtained here

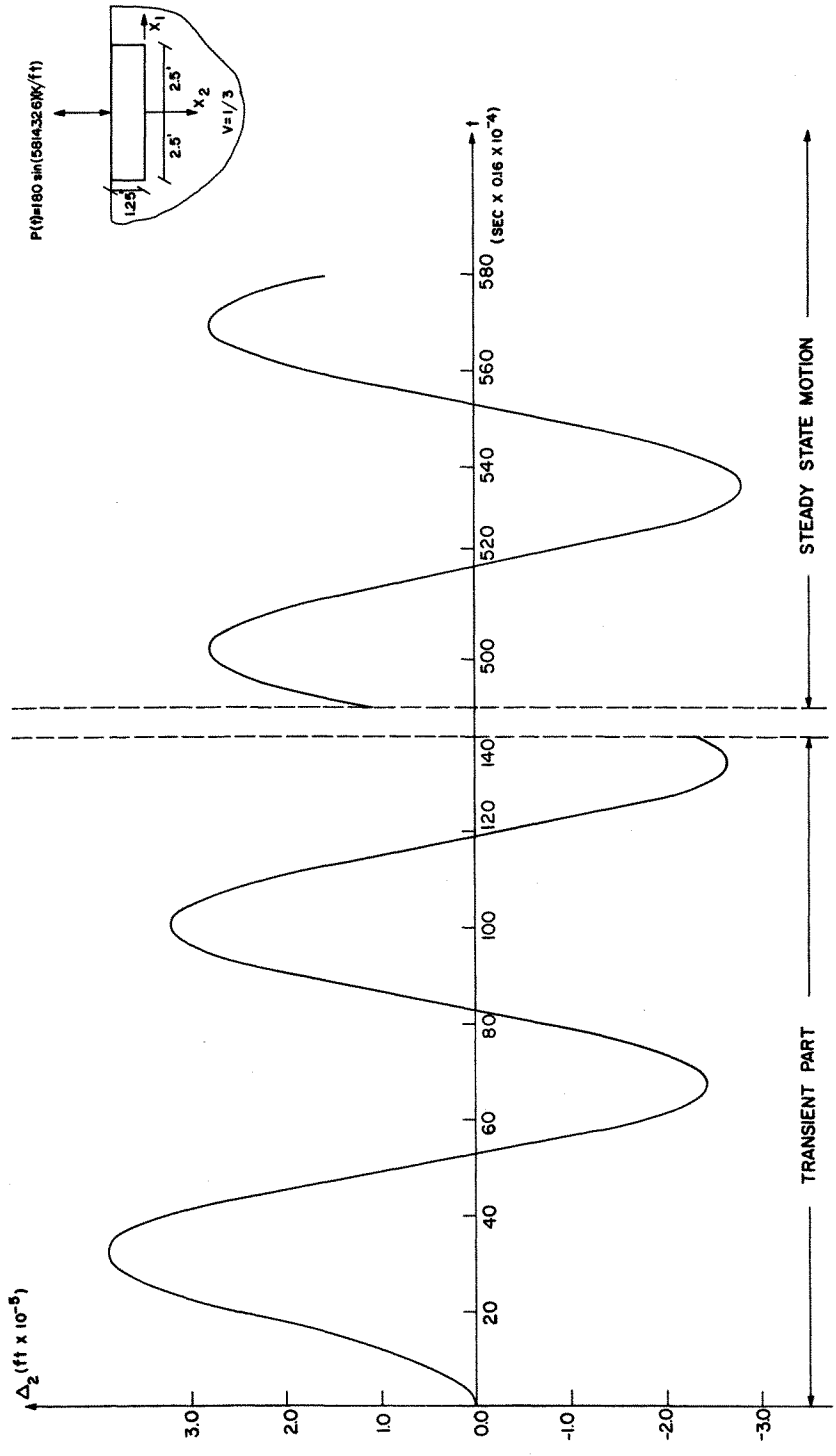

Figure 10. Vertical harmonic force response

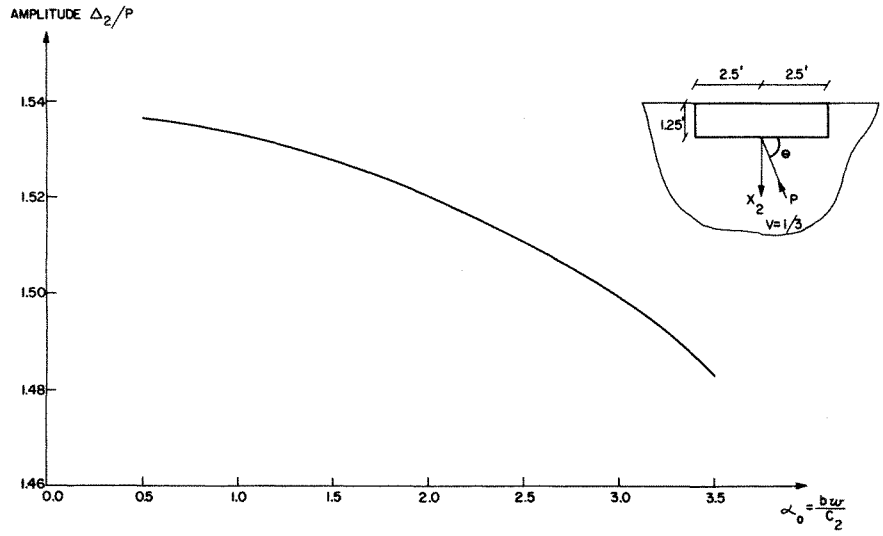

Figure 11(a). Vertical harmonic P-wave response amplitude

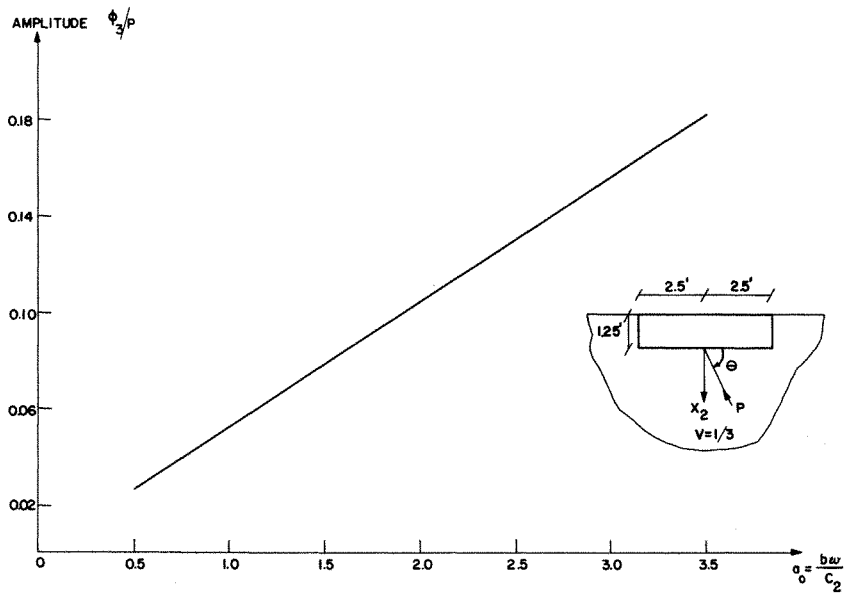

Figure 11(b). Rocking harmonic P-wave response amplitude

for harmonic P-waves. The dimensionless amplitudes of the rigid modes of motion versus the dimensionless frequency of harmonic waves are portrayed in Figures 11(a) and 11(b). More details regarding the computational schemes can be found in reference 42.

CONCLUSIONS

A direct time domain BEM suitable for the solution of general two-dimensional linear elastodynamic problems has been presented and applied for the first time to the solution of two

representative soil–structure interaction problems, namely the dynamic analysis of two-dimensional rigid surface or embedded foundations under conditions of plane strain. In this method the response is obtained by an impulse response superposition approach. The proposed method presents distinct advantages over other numerical methods because it combines the advantages of the BEM as well as a time domain formulation and solution of the problem. Thus,

1. It requires discretization only along the boundary, in contrast to the finite element method (FEM) and the finite difference method (FDM), which require discretization of both the boundary and the interior of the body.
2. It is very well suited for structures of infinite extent, since it automatically accounts for the radiation condition, thereby eliminating the need for extensive discretization or non-reflecting boundaries as in the case of the FEM and the FDM.
3. It determines the response in a direct and natural way and not in two steps involving solution in the frequency domain and Fourier synthesis, as frequency domain methods require.
4. It can handle transient forces or seismic disturbances without any difficulty and also, in contrast to the frequency methods, detect the transient phenomena during early response times preceding the steady-state motion.
5. The present time domain BEM forms a basis for an extension to the case of non-linear soil behaviour, which is impossible by frequency domain methods.

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